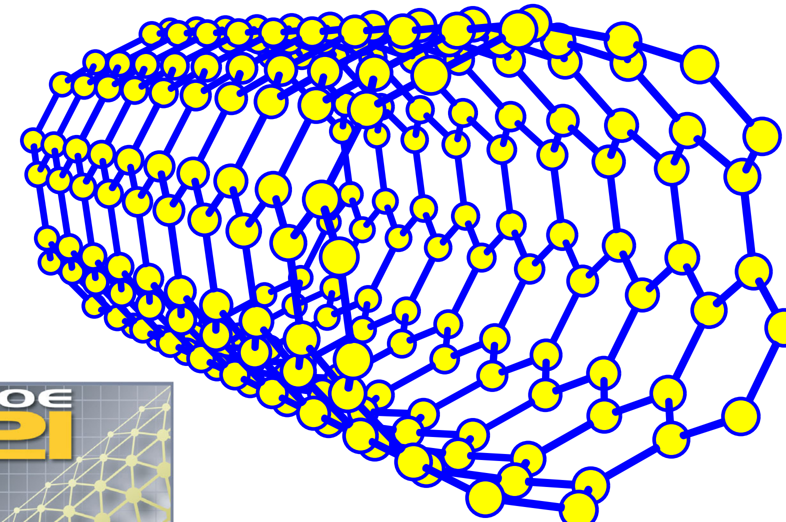
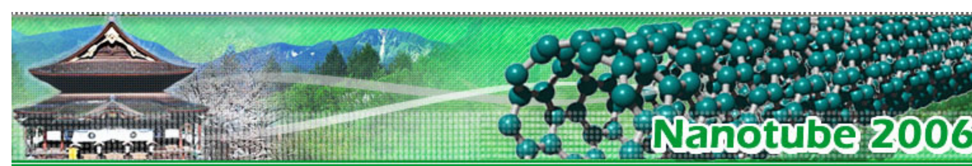
# Metallic Nanotubes as a Perfect Conductor

Tsuneya ANDO

1. **Effective-mass description**
   - Neutrino on cylinder surface
2. **Nanotube as a perfect conductor**
   - Absence of backward scattering
   - Perfectly transmitting channel
   - Some experiments
3. **Effects of symmetry breaking**
   - Inelastic scattering
   - Magnetic field and flux
   - Short-range scatterers
   - Trigonal warping
4. **Inter-wall interaction**
   - Negligible inter-wall conductance
5. **Summary and conclusion**



Sumio Iijima (1991)



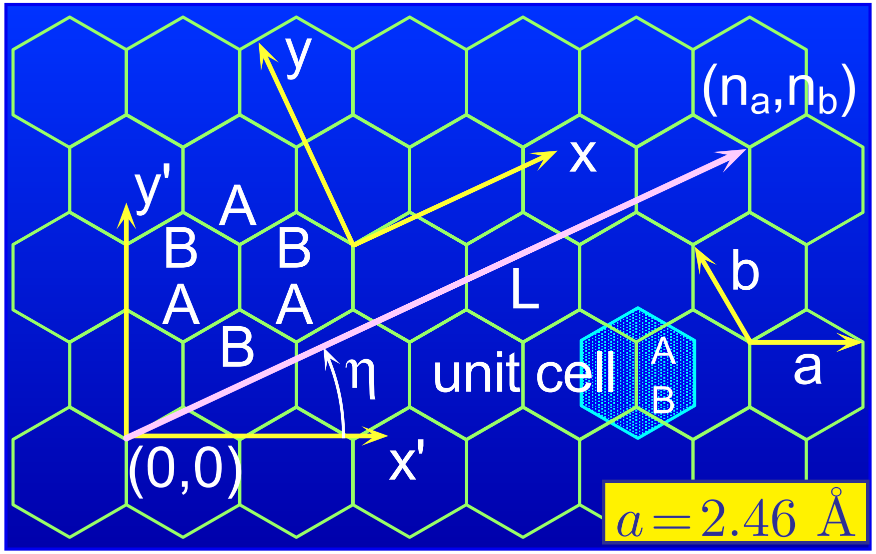

**Collaborators**

T. Nakanishi (AIST)
H. Suzuura (Hokkaido Univ)
S. Uryu (Tokyo Tech)

*Nagano, June 23 (Fri), 2006*

NT06: Seventh International Conference on the Science and Application of Nanotubes, Hotel Metropolitan Nagano, Nagano, Japan, June 18–23, 2006 [09:45-10:15 (25+5)]

# Two-Dimensional Graphite and Carbon Nanotubes




**Chiral vector** : $\boldsymbol{L} = n_a \boldsymbol{a} + n_b \boldsymbol{b}$

**Chiral angle** : $\eta$

## Weyl's equation for neutrinos

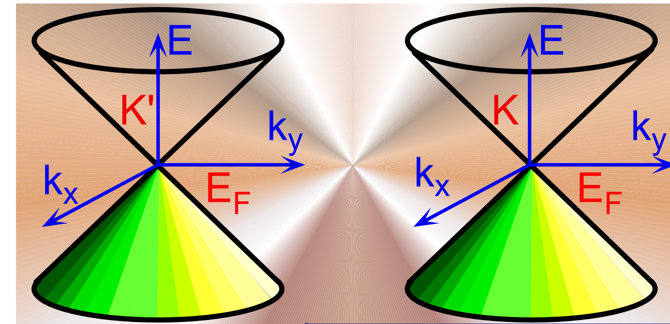
$$\gamma \begin{bmatrix} 0 & \hat{k}_x - i\hat{k}_y \\ \hat{k}_x + i\hat{k}_y & 0 \end{bmatrix} \begin{bmatrix} F^A(\boldsymbol{r}) \\ F^B(\boldsymbol{r}) \end{bmatrix} = \varepsilon \begin{bmatrix} F^A(\boldsymbol{r}) \\ F^B(\boldsymbol{r}) \end{bmatrix}$$

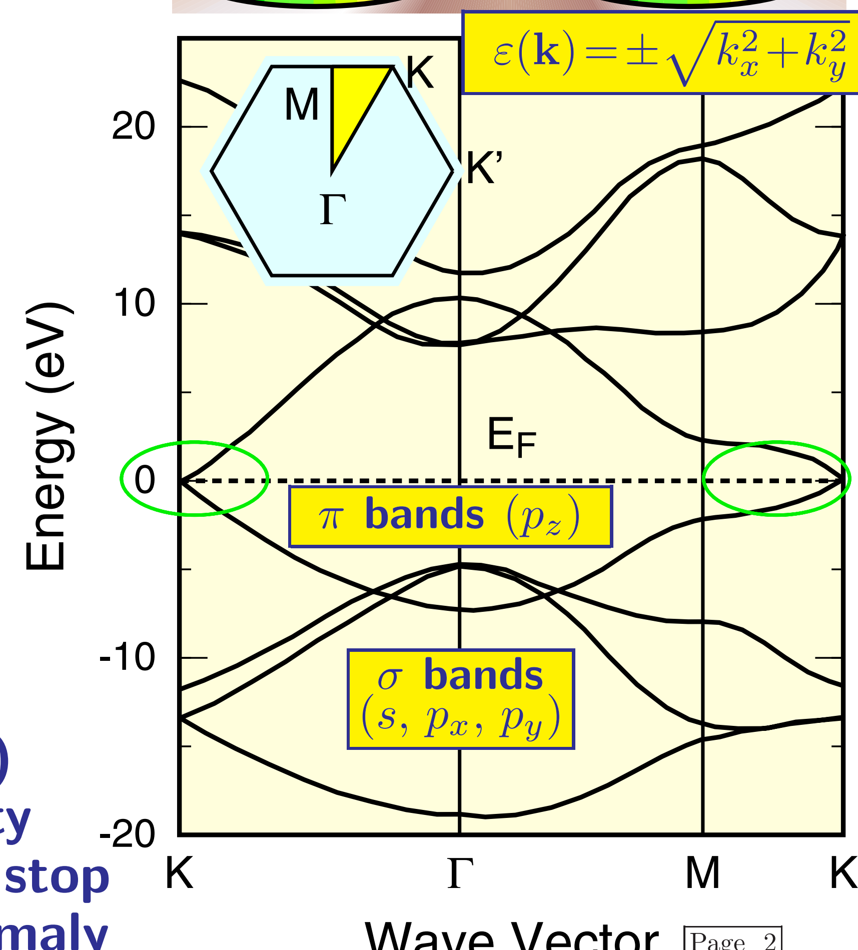
$$\Leftrightarrow \quad \gamma(\sigma_x \hat{k}_x + \sigma_y \hat{k}_y)\boldsymbol{F}(\boldsymbol{r}) = \varepsilon \boldsymbol{F}(\boldsymbol{r})$$

$$\Leftrightarrow \quad \gamma(\vec{\sigma}\cdot\hat{\boldsymbol{k}})\boldsymbol{F} = \varepsilon \boldsymbol{F}$$

**Massless (Dirac)**
**Constant velocity**
**~light, cannot stop**
**Topological anomaly**

$\hat{\boldsymbol{k}} = -i\vec{\nabla}$
**Velocity:** $\gamma/\hbar$



# Periodic Boundary Conditions and Band Structure

## Periodic boundary conditions

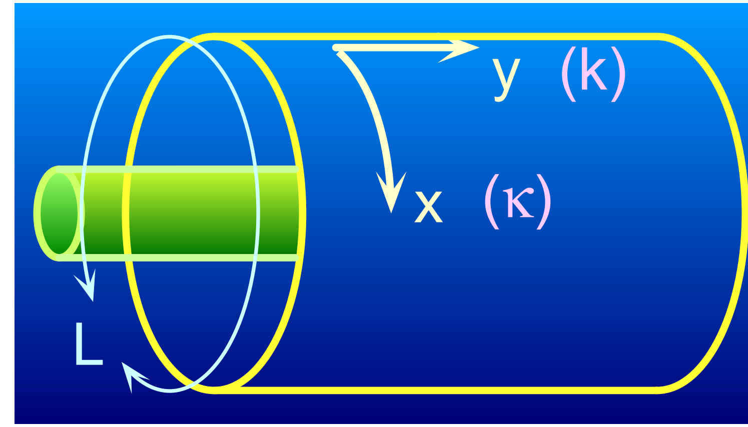
$$\psi(\boldsymbol{r}) = \sum_{j=A,B} F^j(\boldsymbol{r})\Psi_{j\mathrm{K}}(\boldsymbol{r})$$

$$\Psi_{j\mathrm{K}}(\boldsymbol{r}+\boldsymbol{L}) = \Psi_{j\mathrm{K}}(\boldsymbol{r})\exp(i\boldsymbol{K}\cdot\boldsymbol{L})$$

$$\exp(i\boldsymbol{K}\cdot\boldsymbol{L}) = \exp(+2\pi\nu i/3)$$

$$n_a+n_b = 3N+\nu \quad (\nu=0,\pm1)$$

$$\psi(\boldsymbol{r}+\boldsymbol{L}) = \psi(\boldsymbol{r}) \Rightarrow \boxed{\boldsymbol{F}(\boldsymbol{r}+\boldsymbol{L})=\boldsymbol{F}(\boldsymbol{r})\exp(-2\pi\nu i/3)}$$




**Fictitious AB flux**

## Wave functions

$$\boldsymbol{F}(\boldsymbol{r}) \propto \exp[i\kappa_\nu(n)x+iky]$$

$$\kappa_\nu(n) = \frac{2\pi}{L}\left(n-\frac{\nu}{3}\right)$$

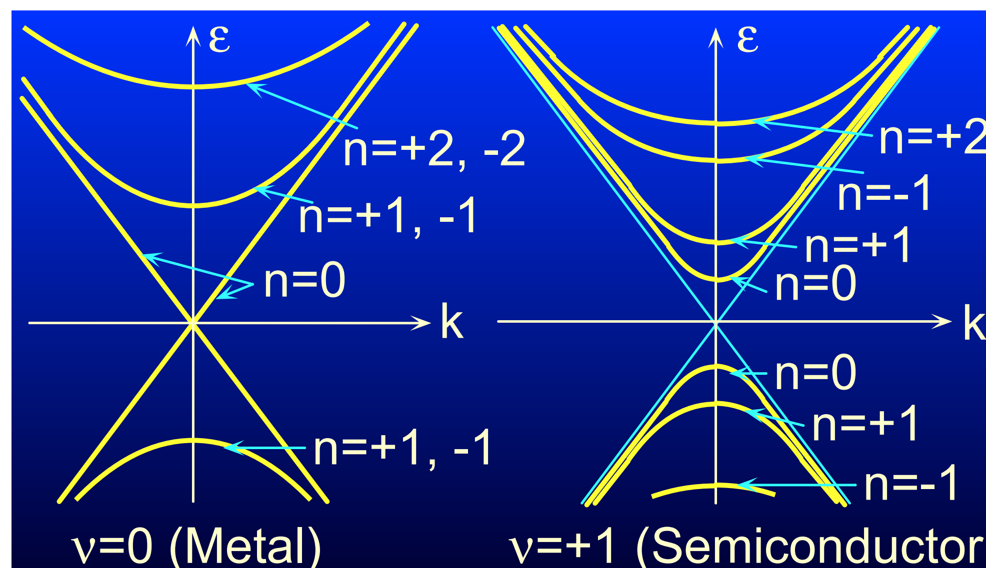
## Energy bands

$$\varepsilon_\nu^{(\pm)}(n,k)=\pm\gamma\sqrt{\kappa_\nu(n)^2+k^2}$$

## Band gap (semiconductor)

$$E_g=4\pi\gamma/3L$$

**K': $\nu \rightarrow -\nu$**




**$k\cdot p$ scheme ⇔ Global and essential properties of CN**

# Topological Anomaly

**Weyl's equation : Neutrino ⇔ Helicity ($\vec{\sigma} \leftrightarrow \boldsymbol{k}$)**

$$\gamma(\vec{\sigma}\cdot\hat{\boldsymbol{k}})\,\boldsymbol{F}_{s\boldsymbol{k}}(\boldsymbol{r}) = \varepsilon_s(\boldsymbol{k})\,\boldsymbol{F}_{s\boldsymbol{k}}(\boldsymbol{r}) \qquad \boldsymbol{F}_{s\boldsymbol{k}}(\boldsymbol{r}) = \frac{1}{\sqrt{LA}}\exp(i\boldsymbol{k}\cdot\boldsymbol{r})\,R^{-1}[\theta(\boldsymbol{k})]\big|s\big)$$

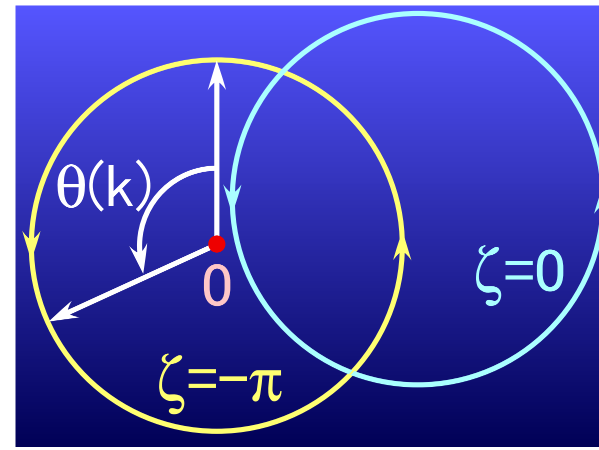
$$\varepsilon_s(\boldsymbol{k}) = s\,\gamma\,|\boldsymbol{k}| \qquad s=\pm 1$$

$$R(\theta\pm 2\pi) = -R(\theta) \qquad R(-\pi) = -R(+\pi)$$

**Pseudo spin ⇒ Berry's phase**

$$R(\theta+2\pi) = e^{-i\zeta}R(\theta)$$

$$\zeta = -i\int_0^T dt\Big\langle s\boldsymbol{k}(t)\Big|\frac{d}{dt}\Big|s\boldsymbol{k}(t)\Big\rangle = -\pi$$



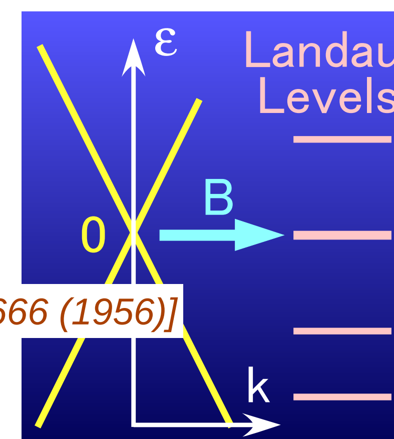

**Absence of backscattering in metallic CNs**

**Perfect conductor in the presence of scatterers**

*T. Ando and T. Nakanishi, JPSJ 67, 1704 (1998)*

**Perfectly conducting channel**

*T. Ando and H. Suzuura, JPSJ 71, 2753 (2002)*




**Landau levels at $\varepsilon=0$ in 2D graphite** *[J.W. McClure PR 104, 666 (1956)]*

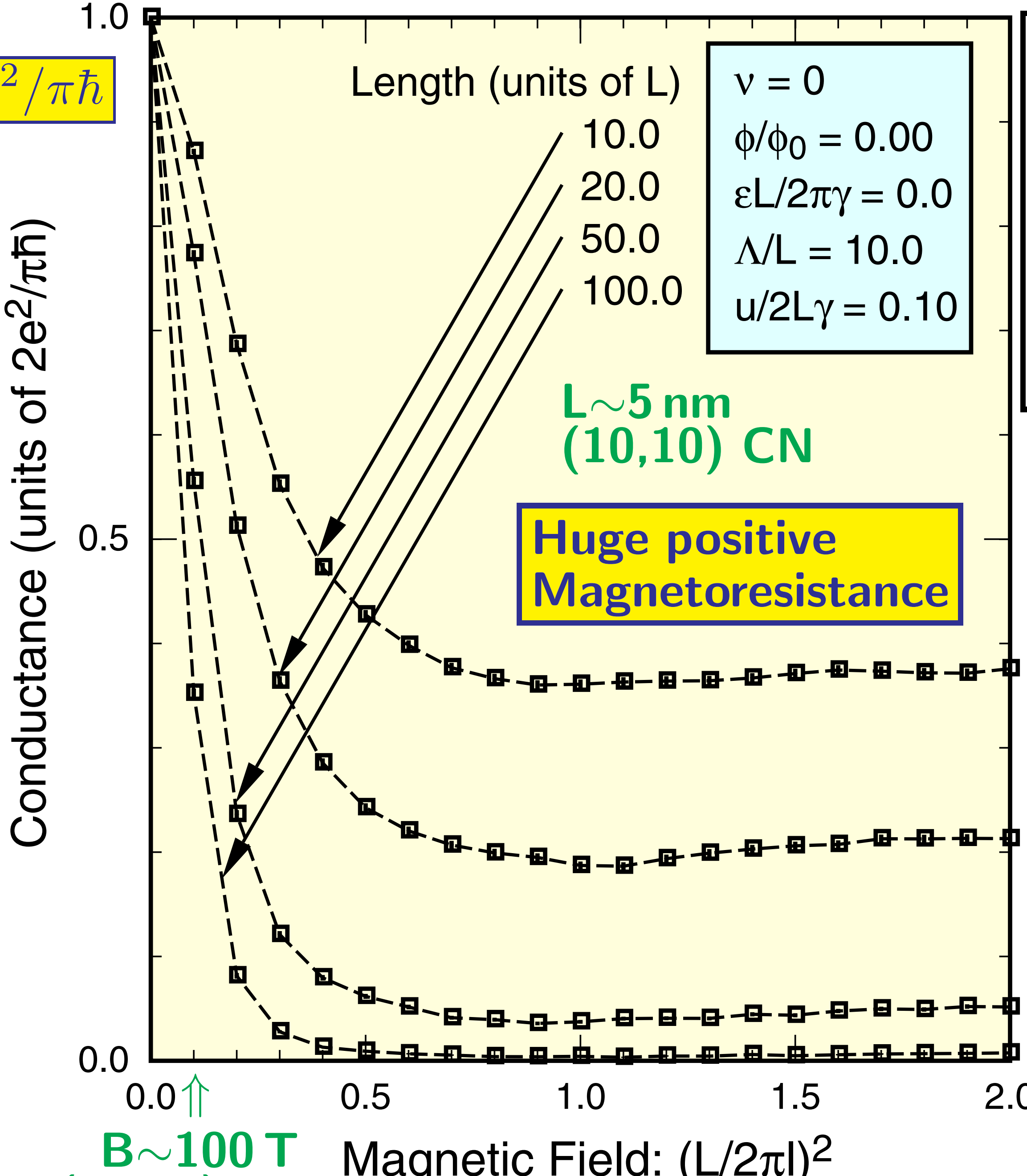
⇒ **Magnetic anisotropy of CN (Field alignment)**

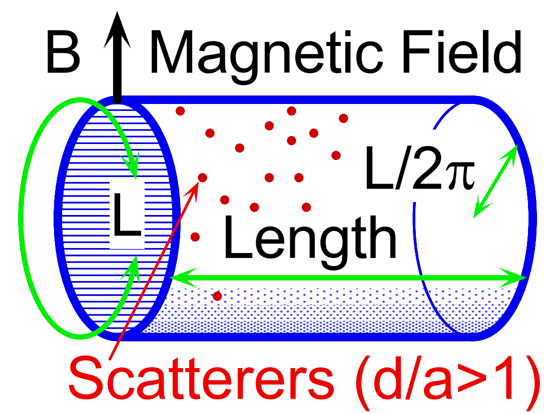
*H. Ajiki and T. Ando, JPSJ 62, 2470 (1993)*

**Experiments:** *S. Zaric et al., Nano Lett. 4, 2219 (2004)*

## Conductance of Finite-Length Nanotubes

*T. Ando and T. Nakanishi, JPSJ 67, 1704 (1998)*




**Nanotube radius**

$R = L/2\pi$

**Magnetic length**

$l = \sqrt{c\hbar/eB}$

**No scatterers with range smaller than lattice constant**

⇒ **Perfect conductor in the presence of scatterers** $(B=0)$

# Helicity, Spin-Rotation, and Berry's Phase

*T. Ando, T. Nakanishi, and R. Saito, J. Phys. Soc. Jpn. 67, 2857 (1998)*

## Weyl's equation : Neutrino ⇔ Helicity ($\vec{\sigma} \leftrightarrow \boldsymbol{k}$)

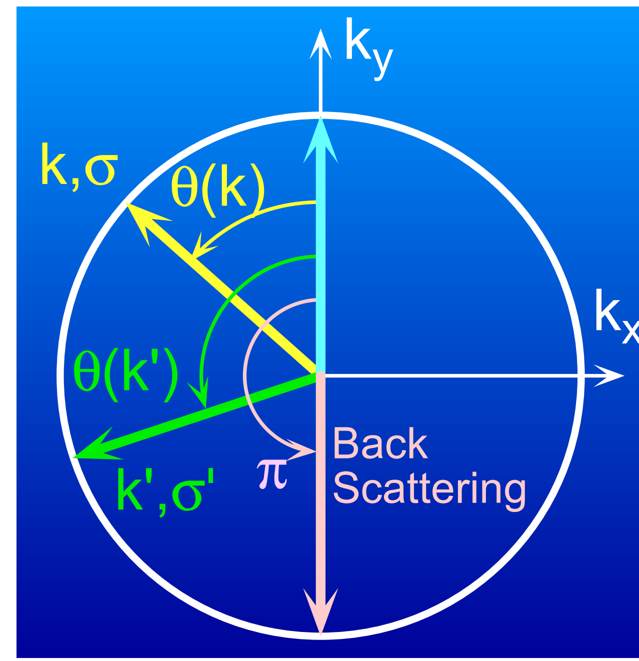
$$\gamma(\vec{\sigma}\cdot\hat{\boldsymbol{k}})\,\boldsymbol{F}_{s\boldsymbol{k}}(\boldsymbol{r}) = \varepsilon_s(\boldsymbol{k})\,\boldsymbol{F}_{s\boldsymbol{k}}(\boldsymbol{r}) \qquad \boldsymbol{F}_{s\boldsymbol{k}}(\boldsymbol{r}) = \frac{1}{\sqrt{LA}}\exp(i\boldsymbol{k}\cdot\boldsymbol{r})\,R^{-1}[\theta(\boldsymbol{k})]\big|s\big)$$

$$\varepsilon_s(\boldsymbol{k}) = s\,\gamma\,|\boldsymbol{k}| \qquad s=\pm 1$$

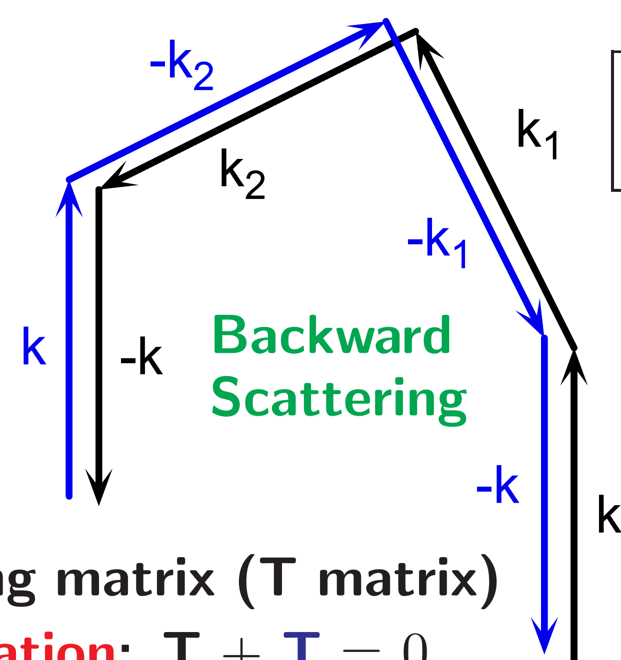
$$R(\theta\pm 2\pi) = -R(\theta) \qquad R(-\pi) = -R(+\pi)$$

$$R(\theta+2\pi) = e^{-i\phi}R(\theta)$$

$\phi$: **Berry's phase (Pseudo-spin)**




**Time reversal path**




$\mathbf{T} \Leftrightarrow +\pi$

$\mathbf{T} \Leftrightarrow -\pi$

$$\phi = -i\int_0^T dt\left\langle s\boldsymbol{k}(t)\left|\frac{d}{dt}\right|s\boldsymbol{k}(t)\right\rangle = -\pi$$

**Scattering matrix (T matrix)**

**Cancellation:** $\mathbf{T} + \mathbf{T} = 0$

**Absence of backward scattering in metallic nanotubes**

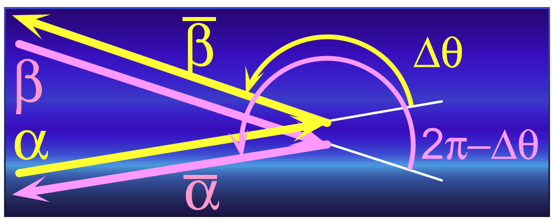
# Multi-Channel Case: Presence of Perfectly Conducting Channel

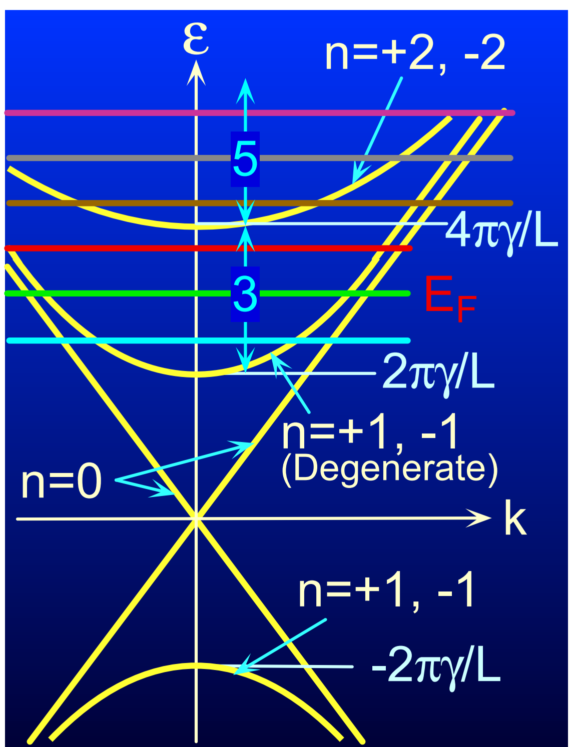
*T. Ando and H. Suzuura, J. Phys. Soc. Jpn. 71, 2753 (2002)*

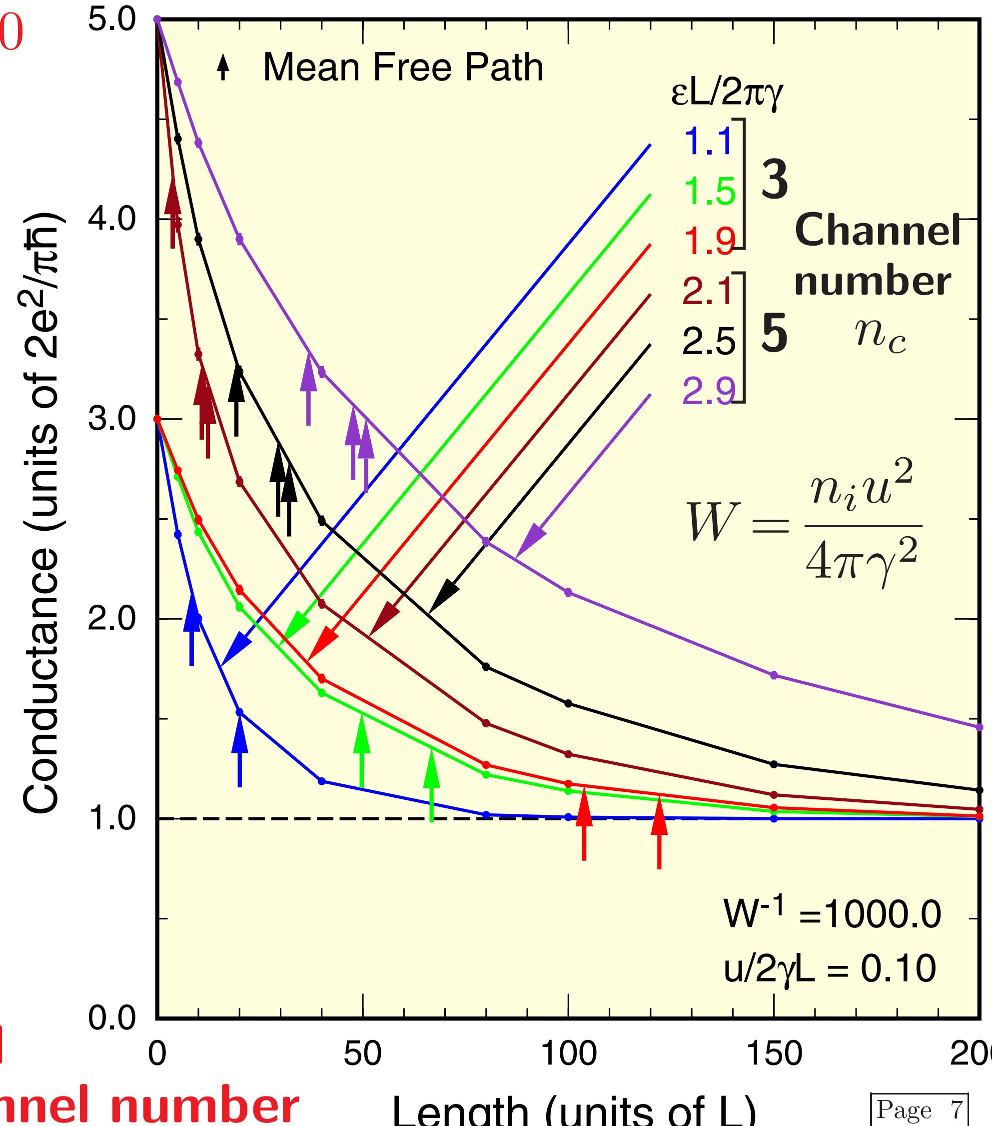
**Time reversal processes:** $\alpha \to \bar{\beta}$ $\Leftrightarrow$ $\beta \to \bar{\alpha}$ $\Rightarrow$ $r_{\bar{\beta}\alpha} = -r_{\bar{\alpha}\beta}$

**Reflection matrix** $\Rightarrow$ $\det(r) = 0$

$\Rightarrow$ **Perfect channel**




**Odd channel number**

# Boltzmann Conductivity and Mean Free Path

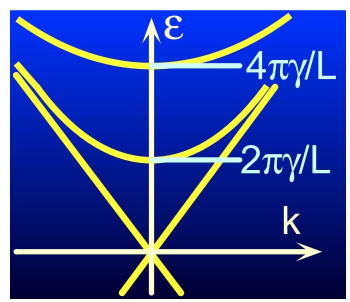
## Transport equation

$$-\frac{e}{\hbar}E\frac{\partial g_{mk}}{\partial k}=\sum_{m'k'}W_{m'k'mk}(g_{m'k'}-g_{mk})$$

$$g_{mk}=f(\varepsilon_{mk})+eEv_{mk}\tau_{mk}\frac{\partial f(\varepsilon_{mk})}{\partial \varepsilon_{mk}}$$

$$\sum_{m'}(K_{m-m'+}-K_{m+m'+})\Lambda_{m'}(\varepsilon)=1$$

$$\Lambda_m(\varepsilon)=|v_{mk}|\tau_{mk}$$




$$K_{\nu\mu}=\frac{A\langle|V_{\nu\mu}|^2\rangle}{\hbar^2|v_\mu v_\nu|}$$

$$\sigma=\int d\varepsilon\left(-\frac{\partial f}{\partial \varepsilon}\right)\sigma(\varepsilon)$$

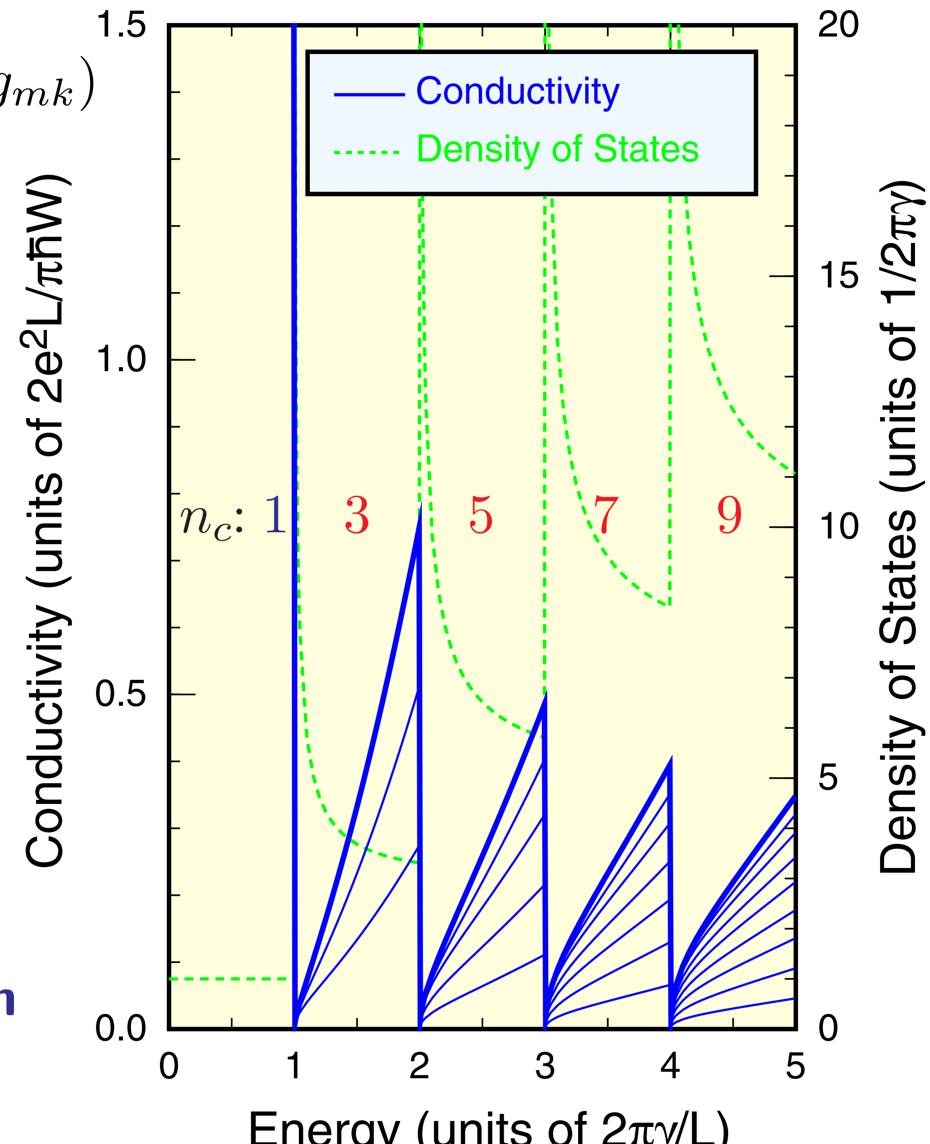
$$\sigma(\varepsilon)=\frac{4e^2}{\pi\hbar}\sum_m \Lambda_m(\varepsilon)$$

⇑

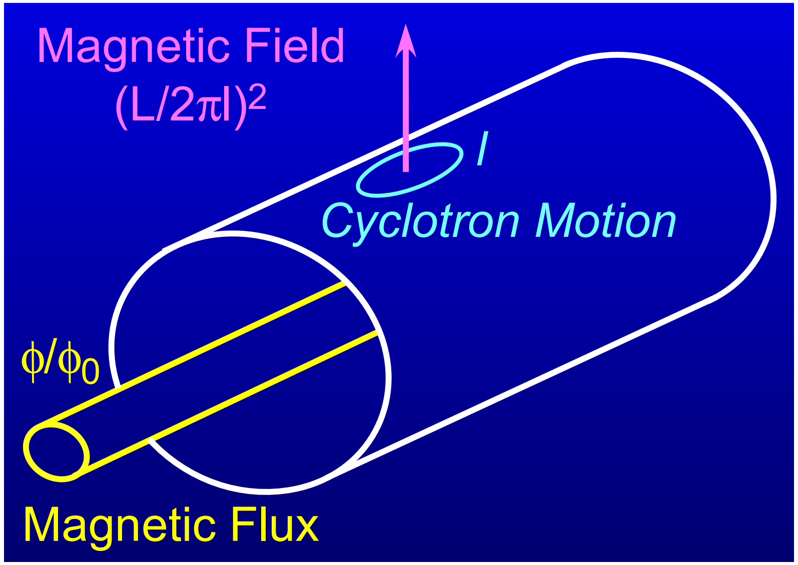
**Conductivity** $\sigma$ **Mean free path**

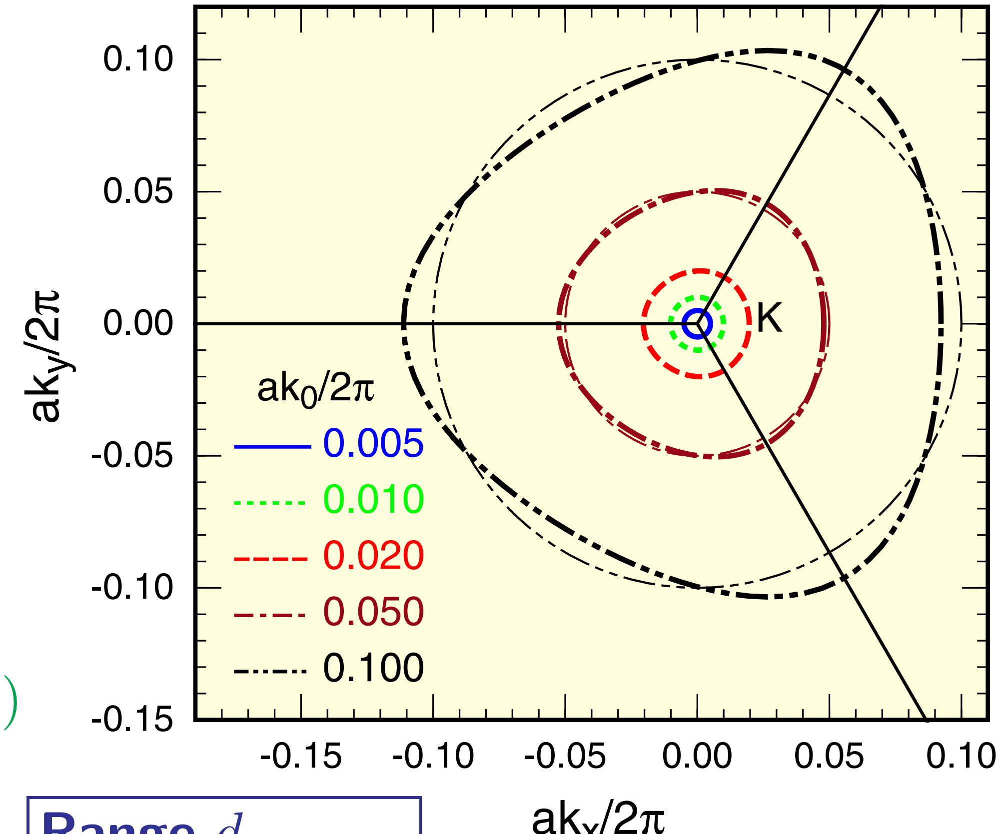
**Infinite** **for** $|\varepsilon|<2\pi\gamma/L$

**Finite** **for** $|\varepsilon|>2\pi\gamma/L$



# Symmetry Breaking Effects




**Magnetic field:** $(L/2\pi l)^2$

**Magnetic flux:** $\phi/\phi_0$ $(\phi_0 = ch/e)$

**Short-range scatterers**

- **Intervalley (K↔K')**
- **'Spin' dependent potential**

**Range** $d$

- **Long :** $d/a \gtrsim 1$
- **Short:** $d/a < 1$

*[H. Ajiki and T. Ando, JPSJ 65, 505 (1996)]*

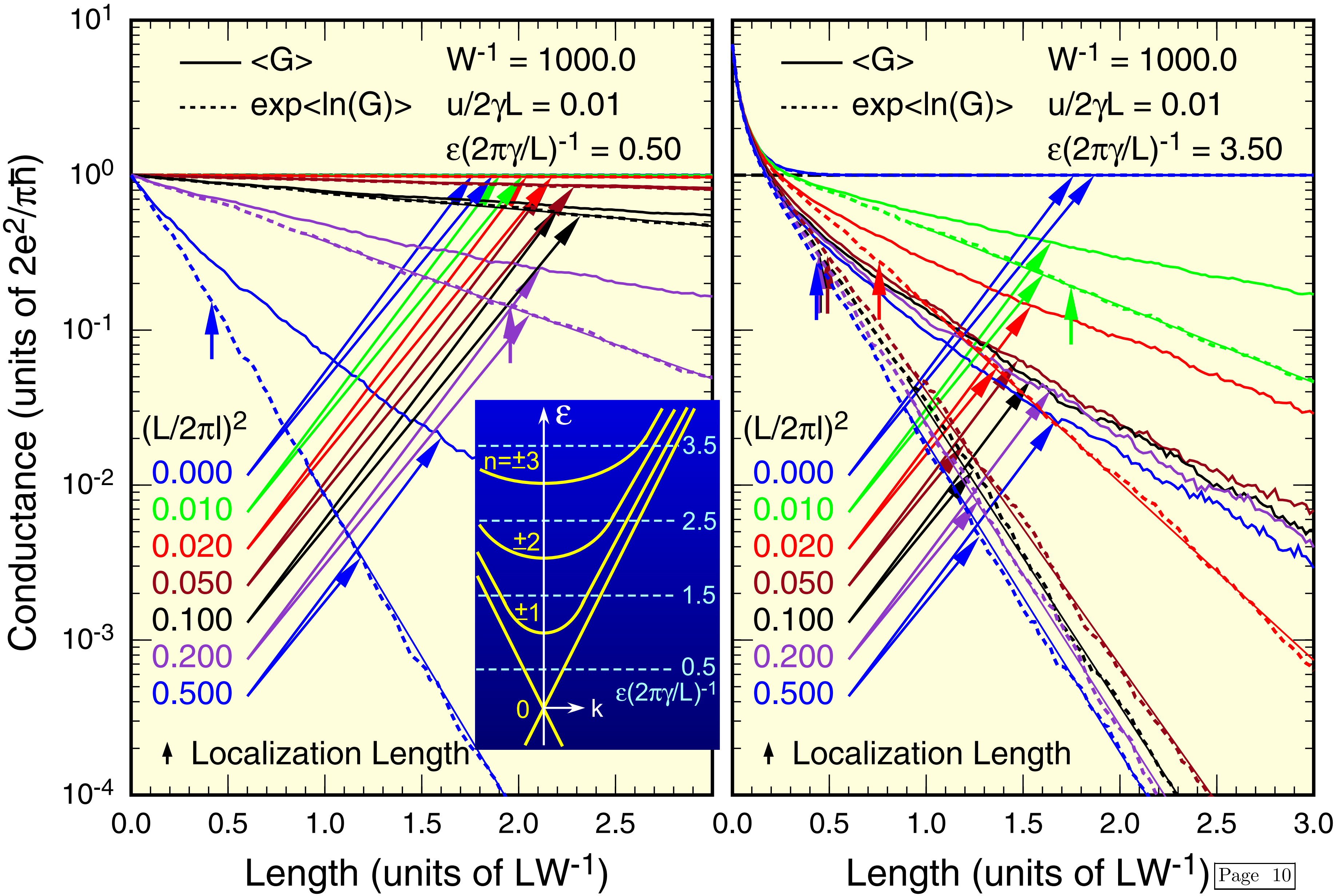
**Trigonal warping** $(\beta \sim 1)$

$$\mathcal{H} = \gamma \begin{pmatrix} 0 & \hat{k}_x - i\hat{k}_y + \frac{\beta a}{4\sqrt{3}} e^{3i\eta} (\hat{k}_x + i\hat{k}_y)^2 \\ \hat{k}_x + i\hat{k}_y + \frac{\beta a}{4\sqrt{3}} e^{-3i\eta} (\hat{k}_x - i\hat{k}_y)^2 & 0 \end{pmatrix}$$

**TB model** $\Rightarrow \beta = 1$

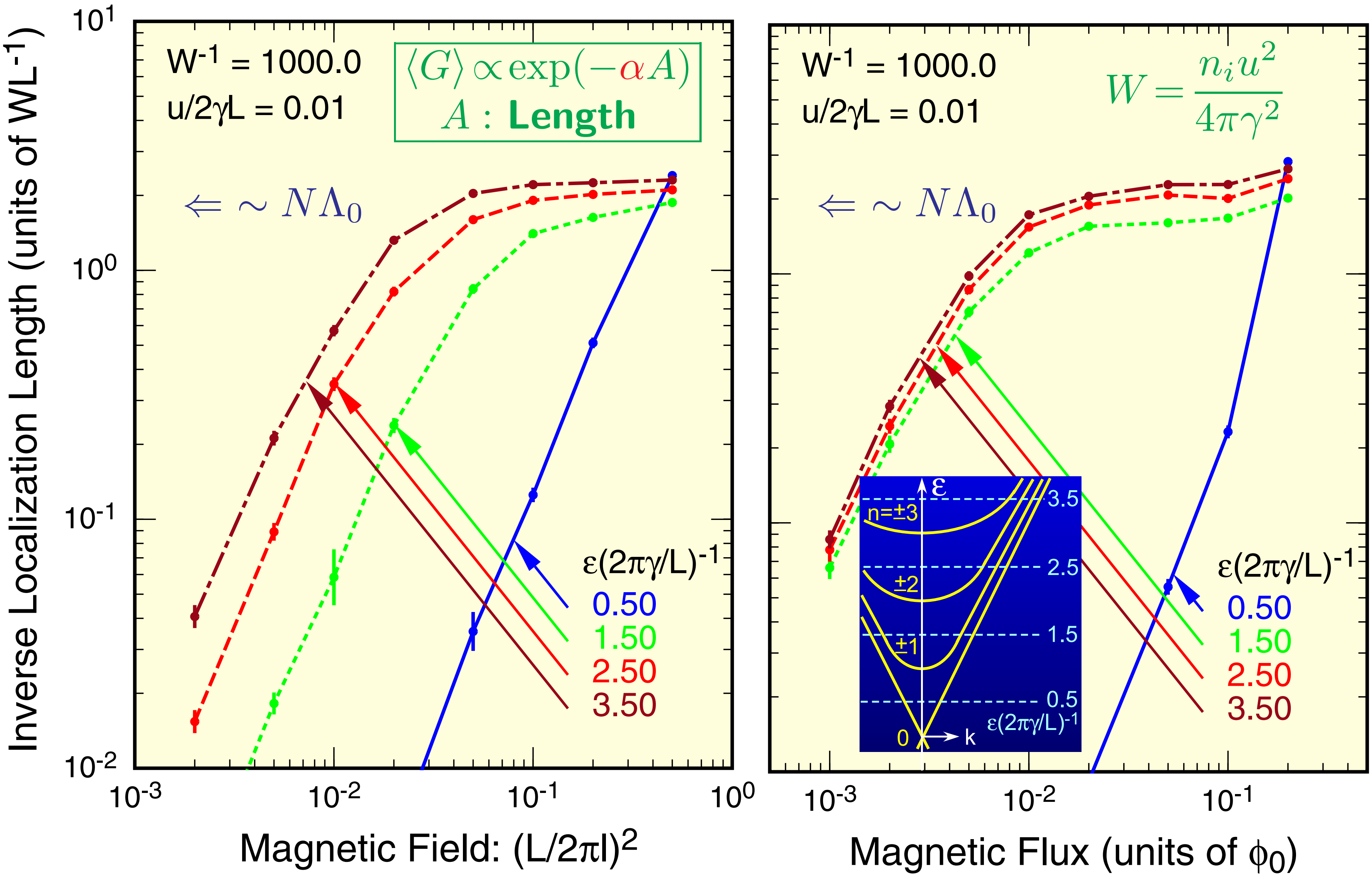
# Conductance in Weak Magnetic Fields

*T. Ando, J. Phys. Soc. Jpn. 73, 1273 (2004)*



# Inverse Localization Length $\alpha$ vs Magnetic Field and Flux

*T. Ando, J. Phys. Soc. Jpn. 73, 1273 (2004)*



# Effective Magnetic Flux

**Curvature:** $\dfrac{\phi}{\phi_0} = -\dfrac{2\pi}{4\sqrt{3}}\dfrac{a}{L}\,p\cos 3\eta$ [*T. Ando, JPSJ 69, 1757 (2000)*]

$$p = 1 - \frac{3}{8}\frac{\gamma'}{\gamma} \ll 1 \quad \gamma = -\frac{\sqrt{3}}{2}V_{pp}^{\pi}a \quad \gamma' = \frac{\sqrt{3}}{2}(V_{pp}^{\sigma} - V_{pp}^{\pi})a$$

*cf. C.L. Kane and E.J. Mele, PRL 78, 1932 (1997)*

**Lattice distortion:** $\dfrac{\phi}{\phi_0} = \dfrac{Lg_2}{2\pi\gamma}[(u_{xx} - u_{yy})\cos 3\eta - 2u_{xy}\sin 3\eta]$

$$u_{xx} = \frac{\partial u_x}{\partial x} + \frac{u_z}{R} \quad u_{yy} = \frac{\partial u_y}{\partial y} \quad u_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$

$L$ : **Circumference**

$R$ : **Radius** $L/2\pi$

$a$ : **Lattice constant**

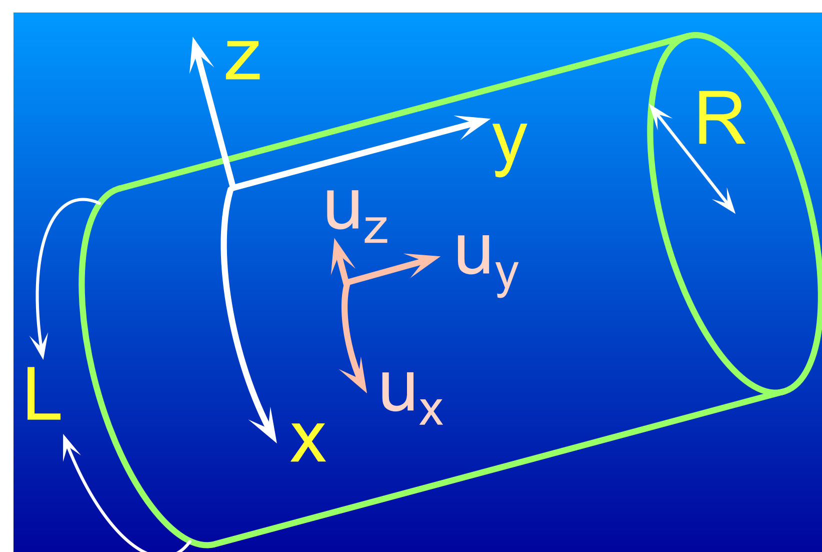
$\eta$ : **Chiral angle**

**Zigzag** : $\eta = 0$

**Armchair:** $\eta = \pi/6$

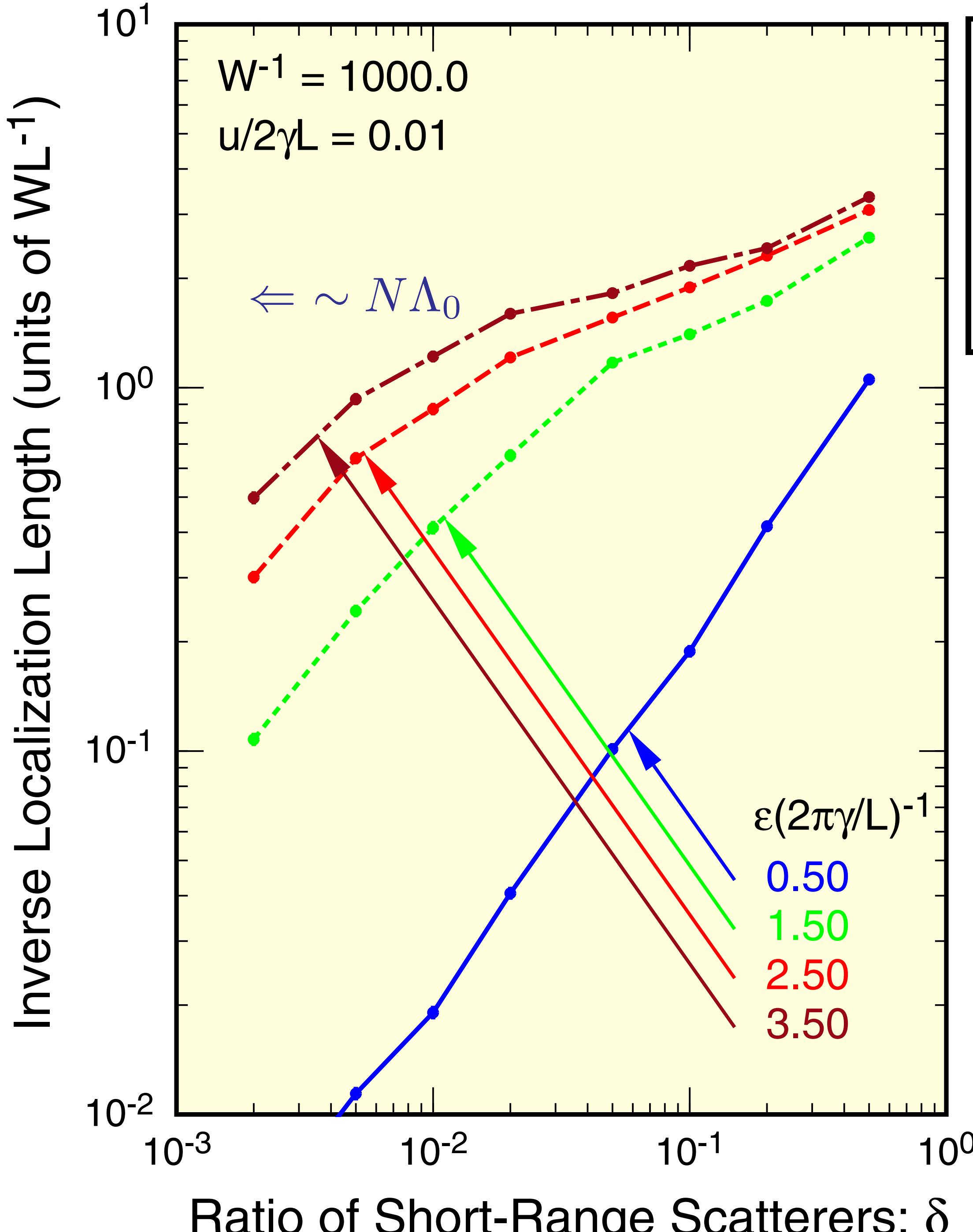
$g_2$ : **Coupling constant** $(\sim |V_{pp}^{\pi}|)$

[*H. Suzuura and T. Ando, PRB 65, 235412 (2002)*]



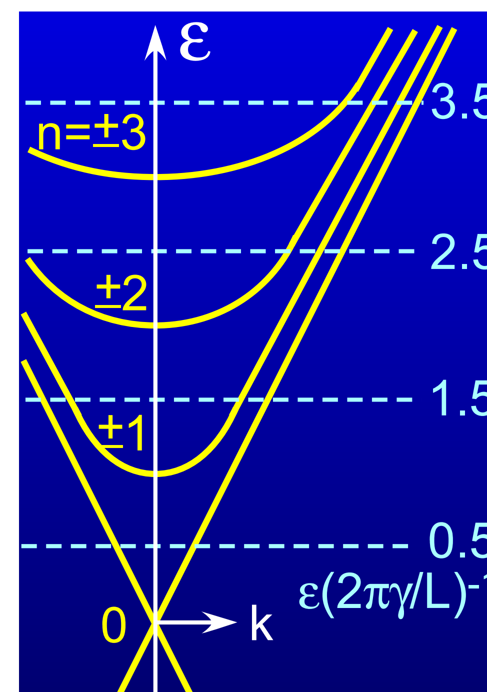
**Effects of Short-Range Scatterers**

*T. Ando and K. Akimoto, JPSJ 73, 1895 (2004)*



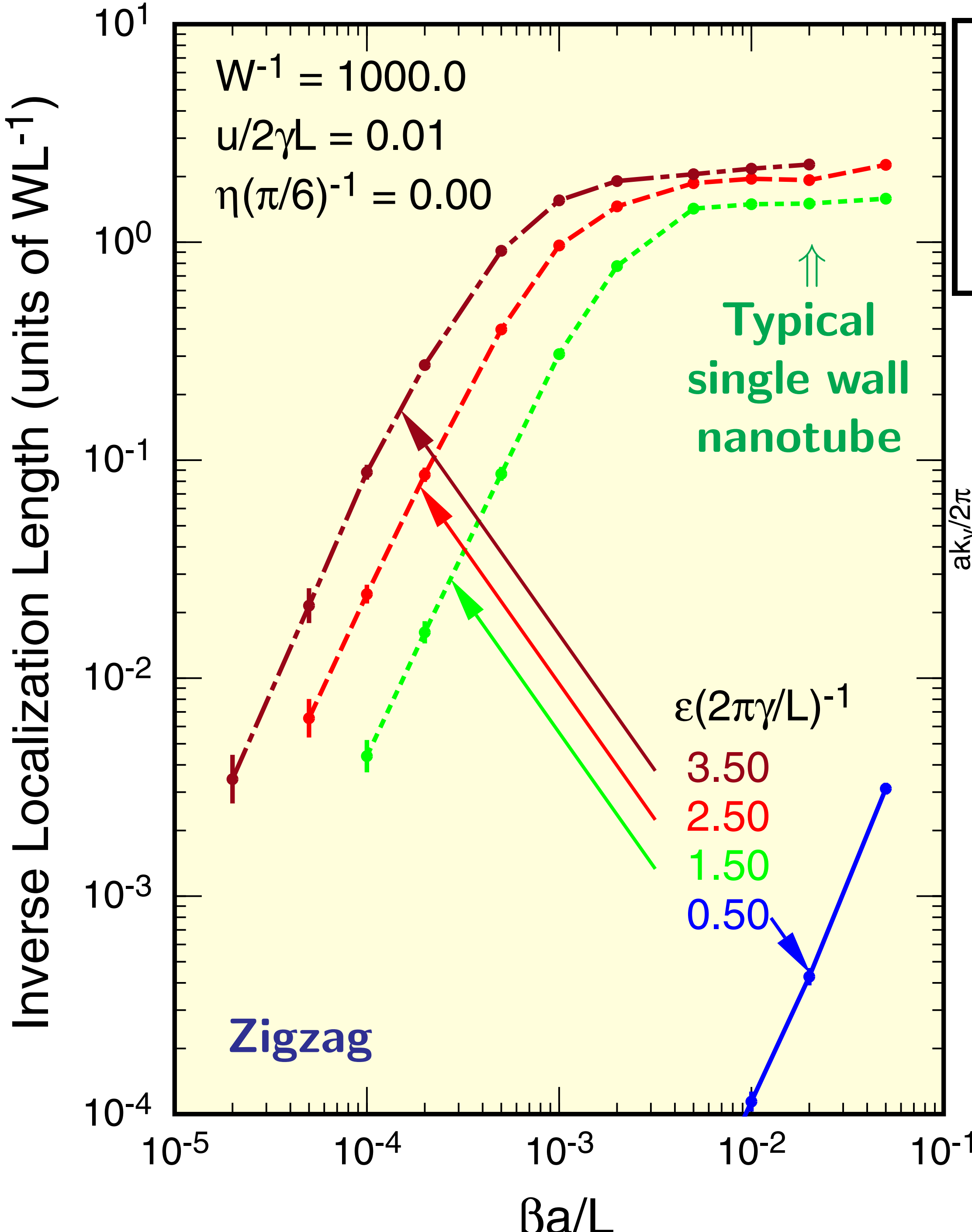

**Scattering strength**

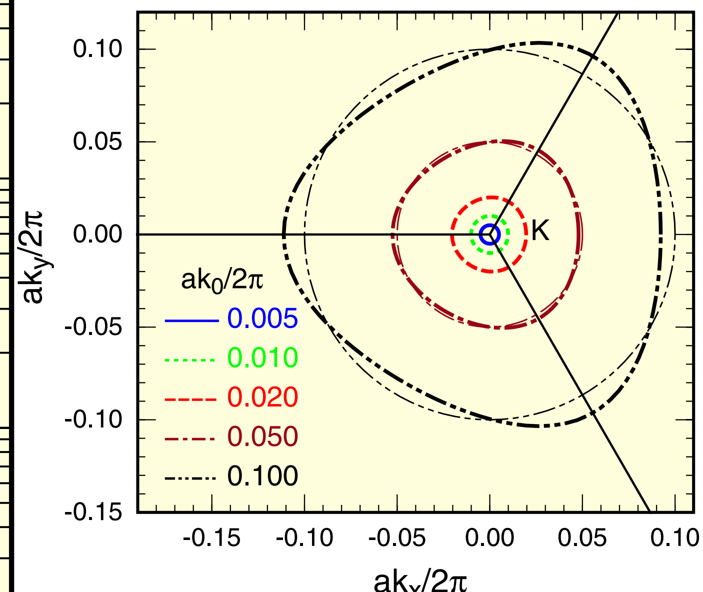
$$W = W_L + W_S$$
$$\delta = W_S / W$$

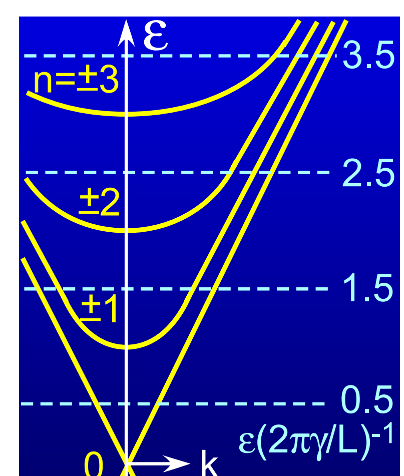


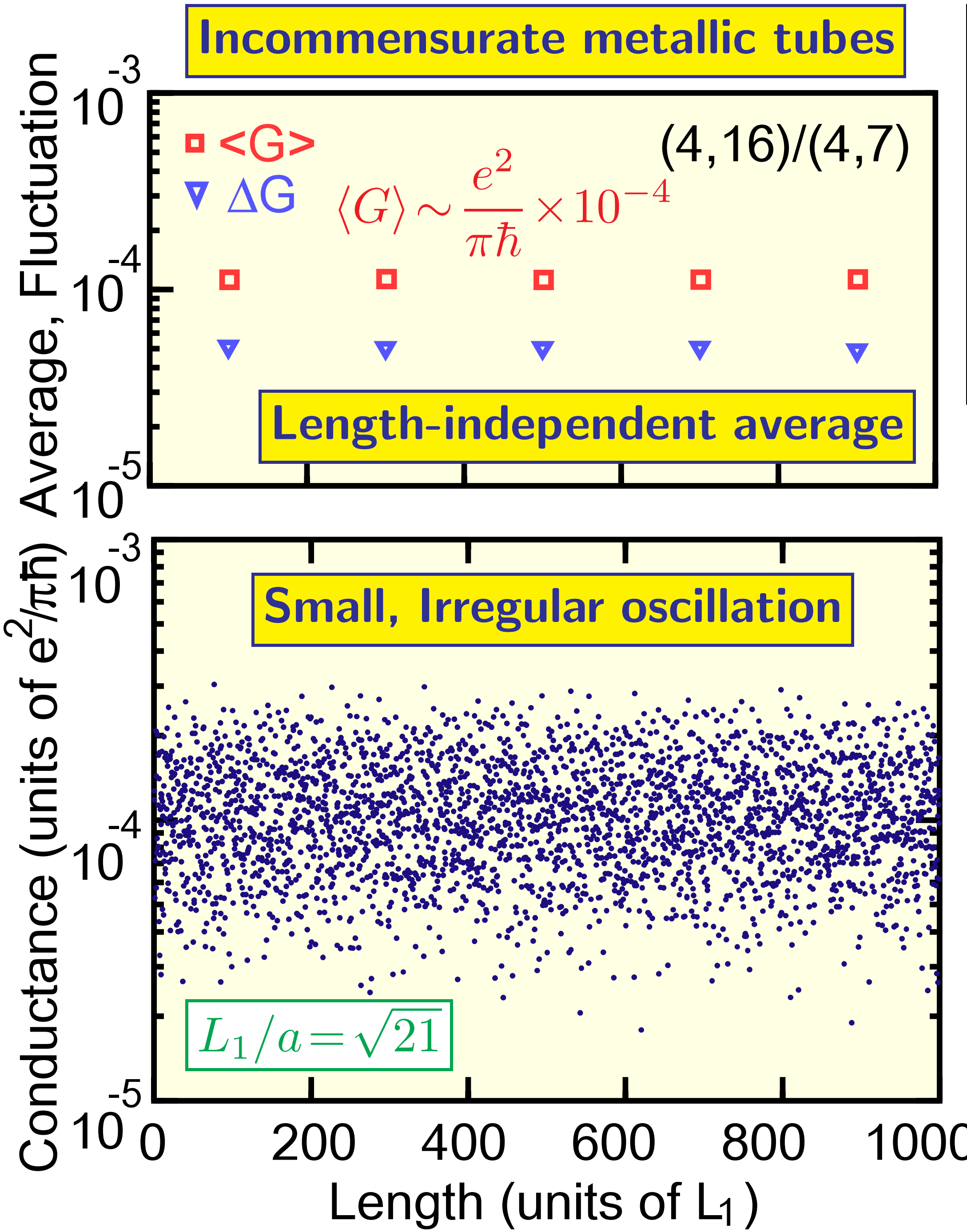
# Effects of Trigonal Warping

*K. Akimoto & T. Ando, JPSJ 73, 2194 (2004)*






**Inter-Wall Conductance of Double-Wall Nanotube**

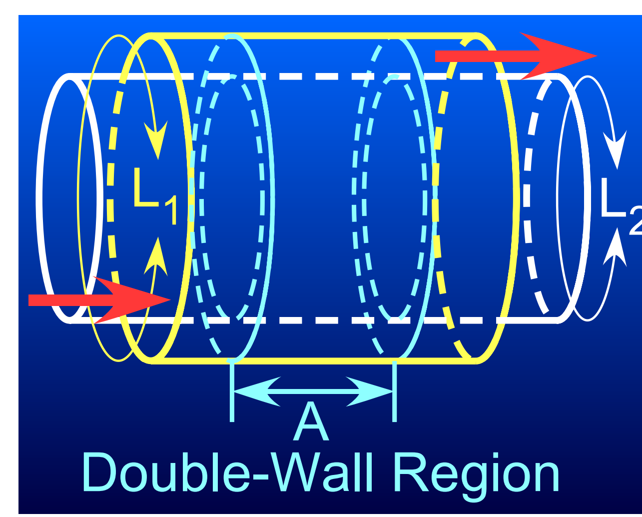
*S. Uryu and T. Ando*

*Phys. Rev. B 72 245403 (2005)*

**Multi-wall nanotubes**

- **Incommensurate lattices**
- **Large inter-wall distance** $\sim$3.6 Å (3.35 Å **Graphite**)

**Four-terminal geometry**



# Inter-Wall Coupling [*S. Uryu and T. Ando, PRB 72, 245403 (2005)*]

**Inter-tube conductance** $\gamma = \frac{\sqrt{3}}{2} a\gamma_0$

$$G = \frac{e^2}{\hbar} \frac{a^2}{4L_1L_2} \frac{1}{\gamma_0^2} \Big| \sum_{\boldsymbol{R}} t(\boldsymbol{R}) \Big|^2$$

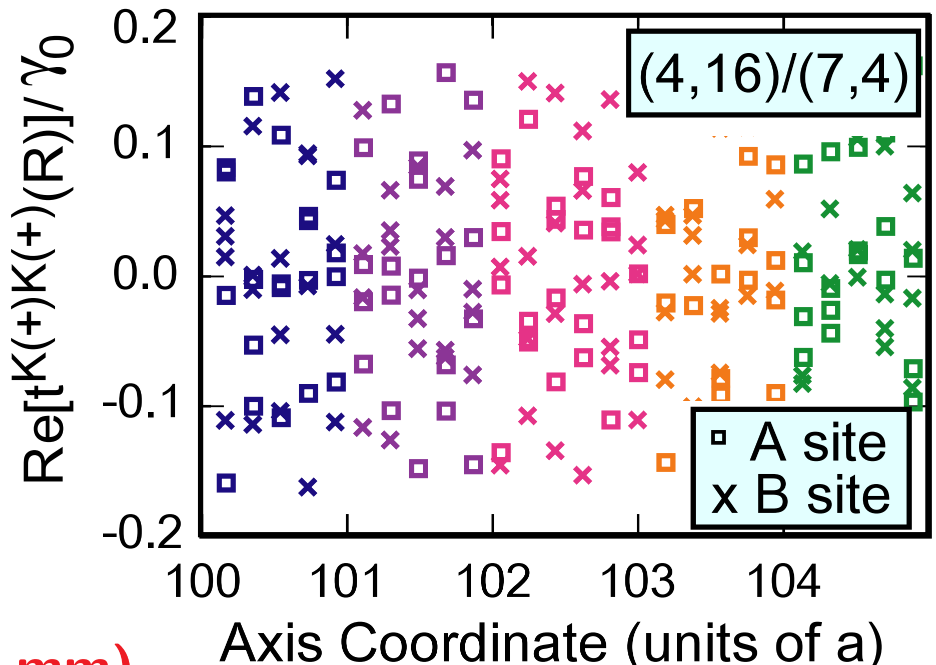
$\approx$ **Series:** $+1, -1, +1, -1, \cdots$

$$\Rightarrow \Big| \sum_{\boldsymbol{R}} t(\boldsymbol{R}) \Big|^2_{\text{typical}} \approx \langle |t(\boldsymbol{R})|^2 \rangle$$

$|t(\boldsymbol{R})| \lesssim 0.1 \times \gamma_0$

$$G \sim 10^{-4} \times \frac{e^2}{\pi\hbar} \quad \Leftarrow (4,16)/(7,4)$$

**Length independent (up to $\sim$1 mm)**



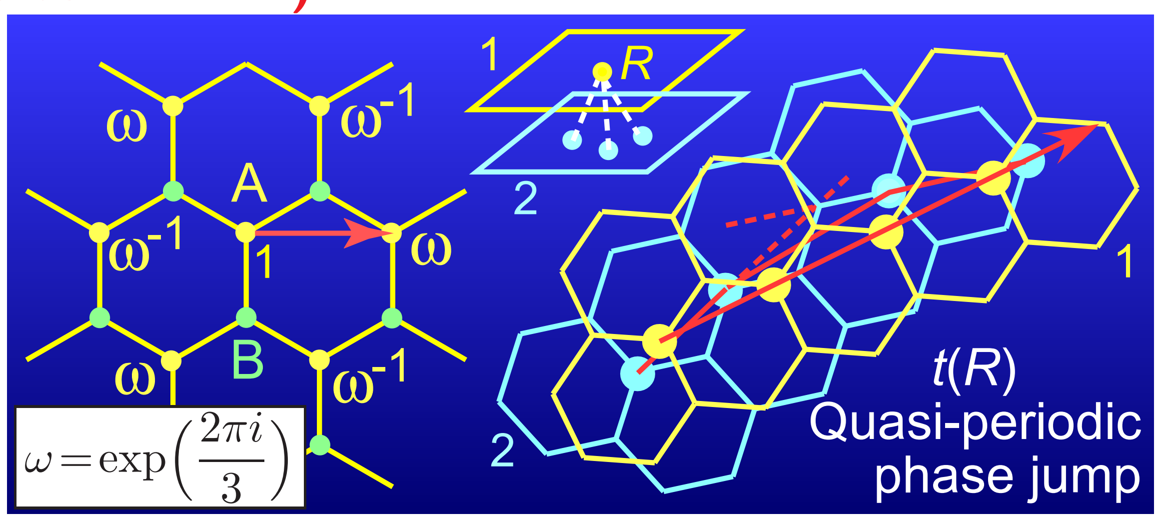

**Some related works**

*Y.-G. Yoon et al., PRB 66, 73407 (2002)*

*K.-H. Ahn et al., PRB 90, 26601 (2003)*

*F. Triozon et al., PRB 69, 121410 (2004)*

*J. Cumings and A. Zettl, PRL 93, 86801 (2004)*



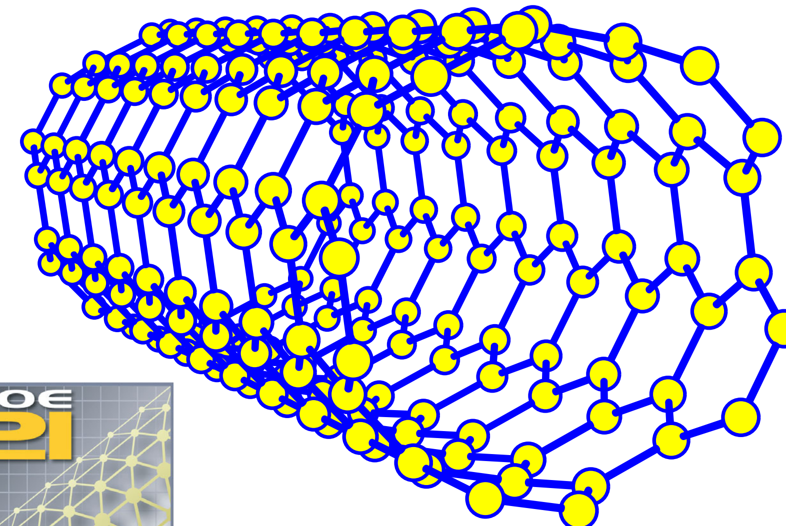
# Summary: Metallic Nanotubes as a Perfect Conductor

Tsuneya ANDO

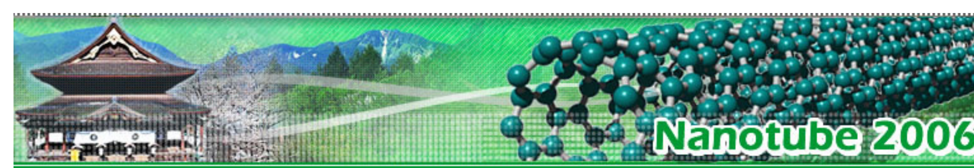
1. Effective-mass description
   - Neutrino on cylinder surface
2. Nanotube as a perfect conductor
   - Absence of backward scattering
   - Perfectly transmitting channel
   - Symmetry and channel number
   - Some experiments
3. Effects of symmetry breaking
   - Inelastic scattering
   - Magnetic field and flux
   - Short-range scatterers
   - Trigonal warping

Absence of backscattering: Robust
Perfect channel : Fragile

4. Inter-wall interaction
   - Negligible inter-wall conductance



Sumio Iijima (1991)




Collaborators
- T. Nakanishi (AIST)
- H. Suzuura (Hokkaido Univ)
- S. Uryu (Tokyo Tech)